Glass Performance Days 2025

10 – 12 JUNE 2025 | NOKIA ARENA - TAMPERE, FINLAND

GLASS PERFORMANCE DAYS 2025

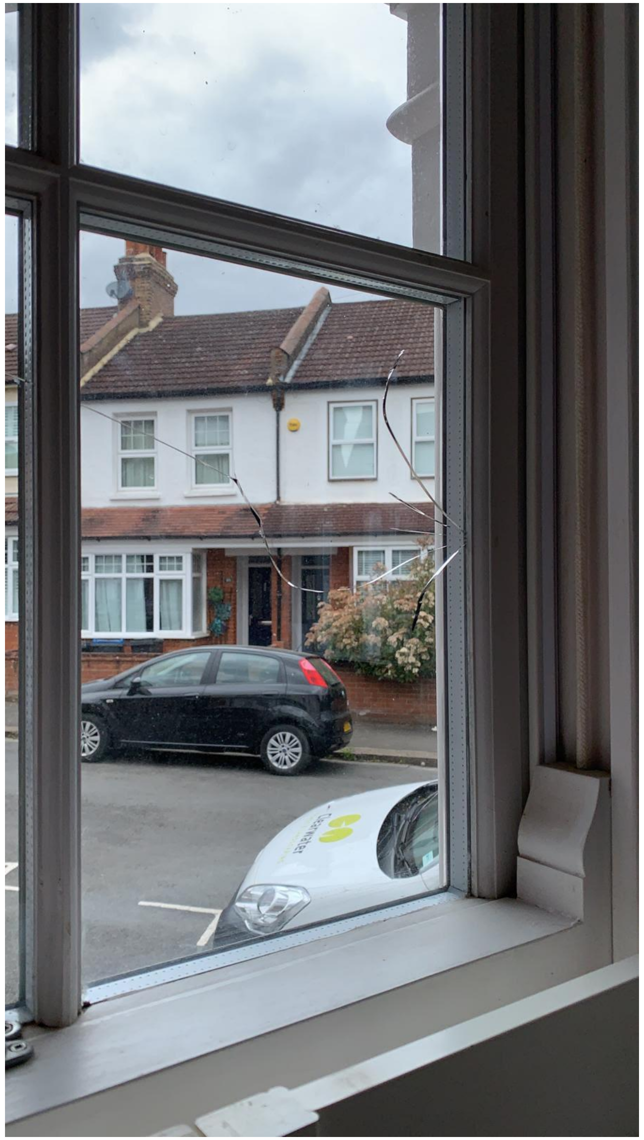
A FLUX-BASED FEM FOR THE THERMAL ANALYSIS OF LAMINATED GLASS FACADES WITH CAST SHADOWS

Maffeis Engineering
Engineering and Consulting

UNIVERSITÀ DI PARMA

Ali Haydar,
Alberto Consolaro,
Massimo Maffeis,
Laura Galuppi,
Gianni Royer Carfagn.

Glass & Thermal Stress



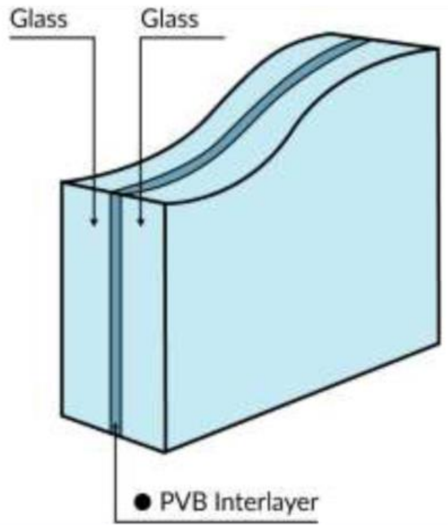
Cast Shadows:

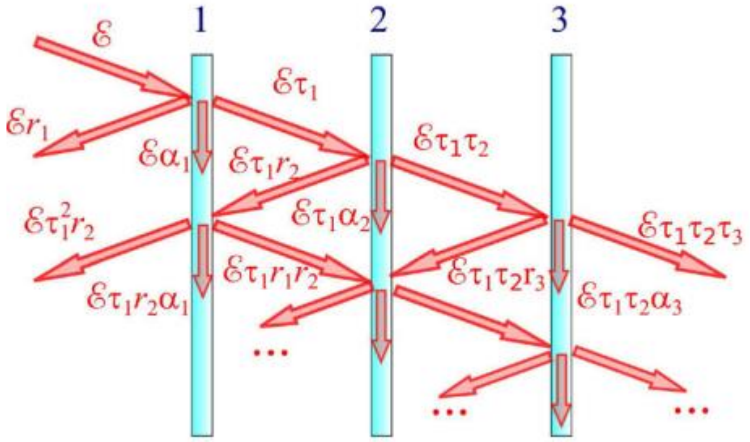
Difficulties:

Non-smooth temperatures field:
- Interface between the different materials
- Interface between shaded and exposed regions

Boundary Conditions:
- Radiation
- Convection

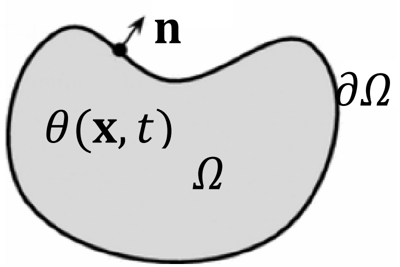
Energy Absorption in Transparent Materials:
- Non-linear Absorption of solar radiation
- Multiple Reflections





Traditional approach

- Temperature $\theta(\mathbf{x},t)$ is the primary variable (scalar field).



- Boundary conditions on heat flux are in terms of the normal gradient of temperature $\nabla\theta(\mathbf{x},t)\cdot\mathbf{n}$, which requires higher continuity for interpolation.

- Temperature field is not regular at the contact surface of different materials.

- Energy conservation is imposed indirectly:

$$c\frac{\partial\theta(x,t)}{\partial t} = \nabla\cdot\left(\lambda\nabla\theta(x,t)\right)$$

- Difficulties arise in modeling non-smooth heating conditions.

Flux Based approach

- The heat displacement vector field is the primary variable (M, Biot 1955) :
  $\dot{\mathbf{H}}(\mathbf{x},t) = -\lambda\nabla\theta(\mathbf{x},t)$,
  λ : thermal conductivity.

- Boundary conditions on heat flux are in terms of the primary variable $\dot{\mathbf{H}}(\mathbf{x},t)\cdot\mathbf{n}$. Linear FE interpolation is sufficient.

- Heat displacement field is continuous.

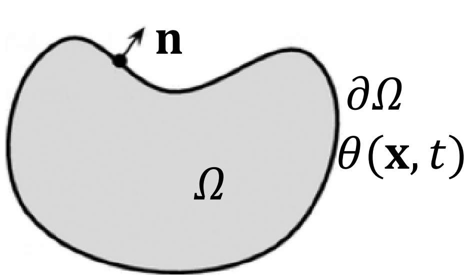
- Energy conservation becomes a holonomic constraint:
  $-\nabla.\mathbf{H}(\mathbf{x},t) = c\theta(\mathbf{x},t)$,
  c : thermal capacity.

- Non-smooth heating conditions (e.g., step changes in heat input) can be handled directly, as the heat displacement field remains well-behaved.

The Neat Flux Based Variational Principle:

- We formulate a neat variational principle, where heat displacement $\mathbf{H}(\mathbf{x}, t)$ is the only variable in the domain, while temperature appears on the boundary.

$$\int_{\Omega} \frac{1}{c} \, \nabla \cdot \mathbf{H}(\mathbf{x}, t) \, \nabla \cdot \delta\mathbf{H} \, dV + \int_{\Omega} \frac{1}{\lambda} \, \dot{\mathbf{H}}(\mathbf{x}, t) \cdot \delta\mathbf{H} \, dV$$
$$= -\int_{\partial\Omega} \theta(\mathbf{x}, t) \delta\mathbf{H} \cdot \mathbf{n} \, ds - \int_{\Omega} \frac{1}{c} \, \nabla \cdot \mathbf{H}^{\#}(\mathbf{x}, t) \, \nabla \cdot \delta\mathbf{H} \, dV + \int_{\Omega} \theta_0(\mathbf{x}) \nabla \cdot \delta H \, dV \, .$$



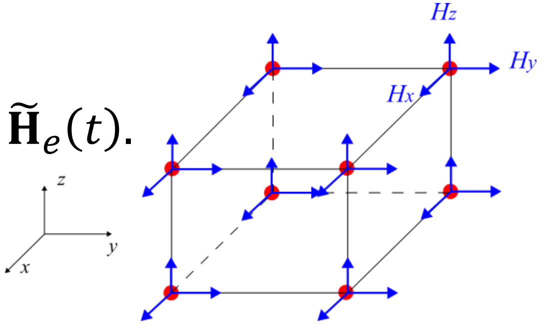
- $\mathbf{H}^{\#}(\mathbf{x}, t)$: Heat displacement field, representative of heat generation & absorption of solar radiation.
- $\theta_0(\mathbf{x})$: initial temperature in the body.
- $\theta(\mathbf{x}, t)$: temperature on the boundary (when and where assigned).

FE:

The neat VP allows a simple FE implementation

Standard Hex8:

$$\mathbf{H}_e(\mathbf{x}, t) = \phi(\mathbf{x})\, \tilde{\mathbf{H}}_e(t).$$



Discretized equations:

$$\mathbf{K}\, \tilde{\mathbf{H}}(t) + \mathbf{C}\, \dot{\tilde{\mathbf{H}}}(t) = q(t) + q_0 - q^{\#}(t).$$

Temperature field is recovered a posteriori

$$\lambda \nabla \theta(\mathbf{x}, t) + \dot{\mathbf{H}}(\mathbf{x}, t) = 0.$$

BC:

Prescribed Temperature at the boundary:

$\theta(\mathbf{x}, t) = \bar{\theta}(\mathbf{x}, t)\ \ \forall \mathbf{x} \in \partial\Omega, \forall t.$ ⟺ Contact with heat reservoir

Prescribed Heat flux at the boundary:

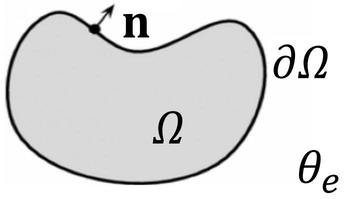
$$\dot{\mathbf{H}}(\mathbf{x}, t) \cdot \mathbf{n} = \bar{q}(\mathbf{x}, t)\ \ \forall \mathbf{x} \in \partial\Omega, \forall t.$$

$\bar{q}(\mathbf{x}, \boldsymbol{t}) = 0$ ⟺ Insulated boundary

Robin Condition:

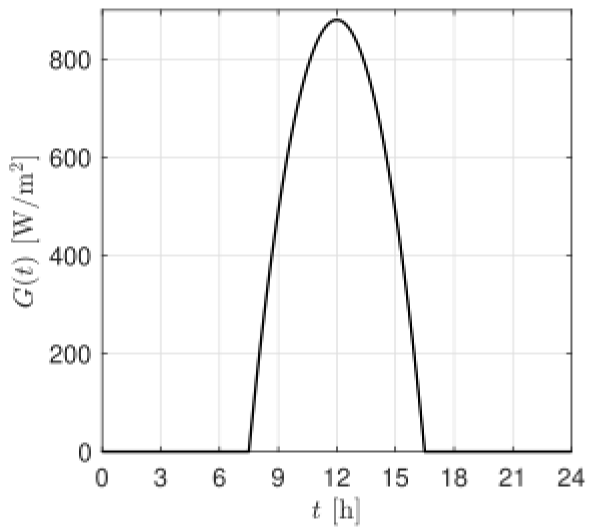
$\dot{\mathbf{H}}(\mathbf{x}, t) \cdot \mathbf{n} = \sigma\epsilon\left[\theta_e(\mathbf{x}, t)^4 - \theta(\mathbf{x}, t)^4\right] \forall \mathbf{x} \in \partial\Omega, \forall t.$ ⟺ Radiation

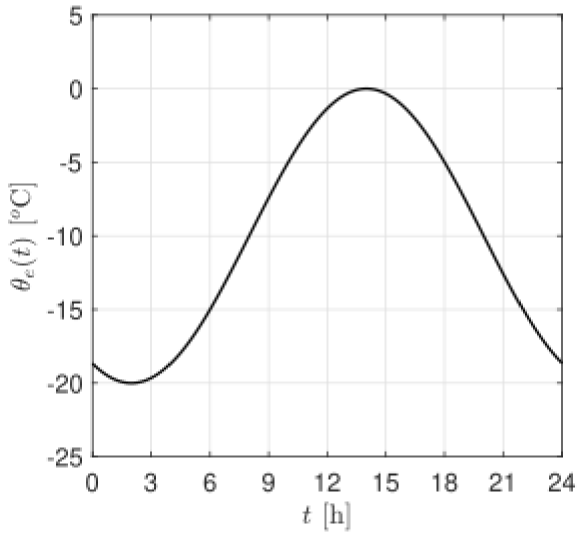
$\dot{\mathbf{H}}(\mathbf{x}, t) \cdot \mathbf{n} = h[\theta_e(\mathbf{x}, t) - \theta(\mathbf{x}, t)]\ \forall \mathbf{x} \in \partial\Omega, \forall t.$ ⟺ Convection (Linearized radiation)



Problem data

- Partially shaded laminated glass plate.
- Shading factor 10% of solar radiation
- Time dependent solar radiation and external temperature.
- Various (possibly varying) shadow shapes.
- Conduction & Convection & Radiation.

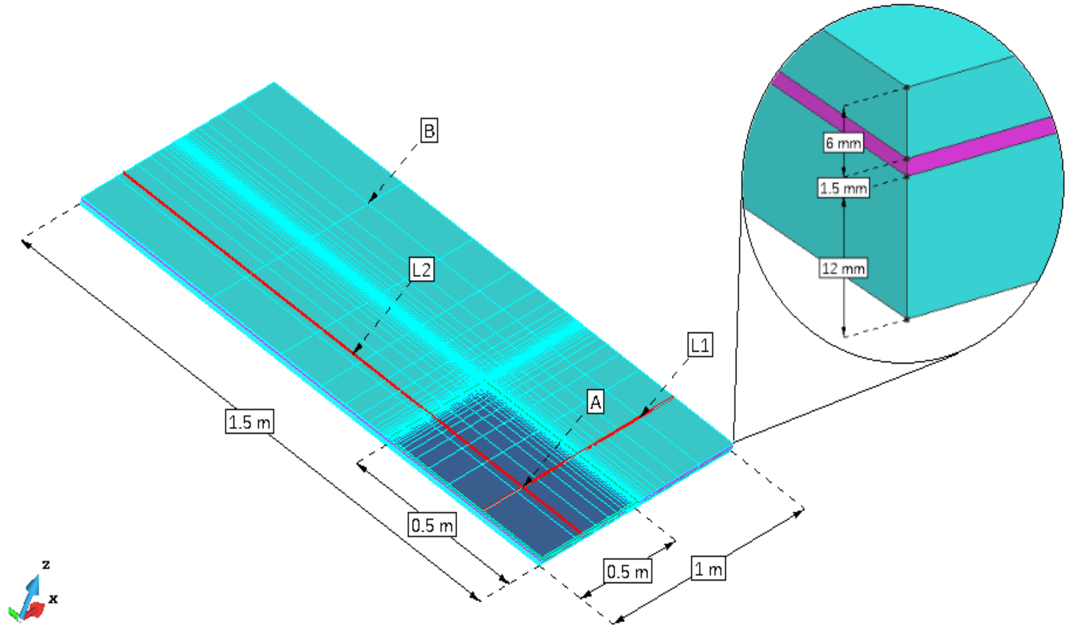




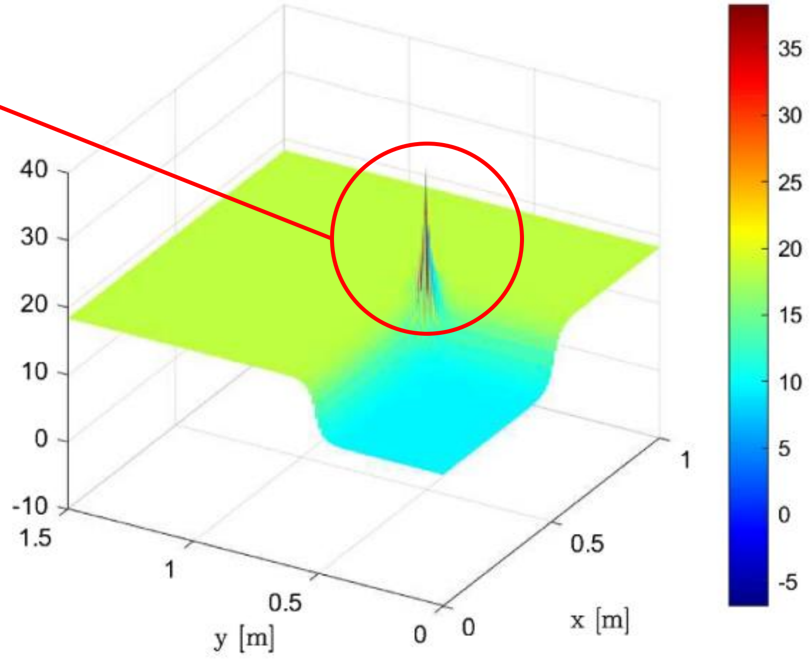
Time Variation of Solar radiation and external Temperature (winter scenario)

Ex.1: Square Shadow

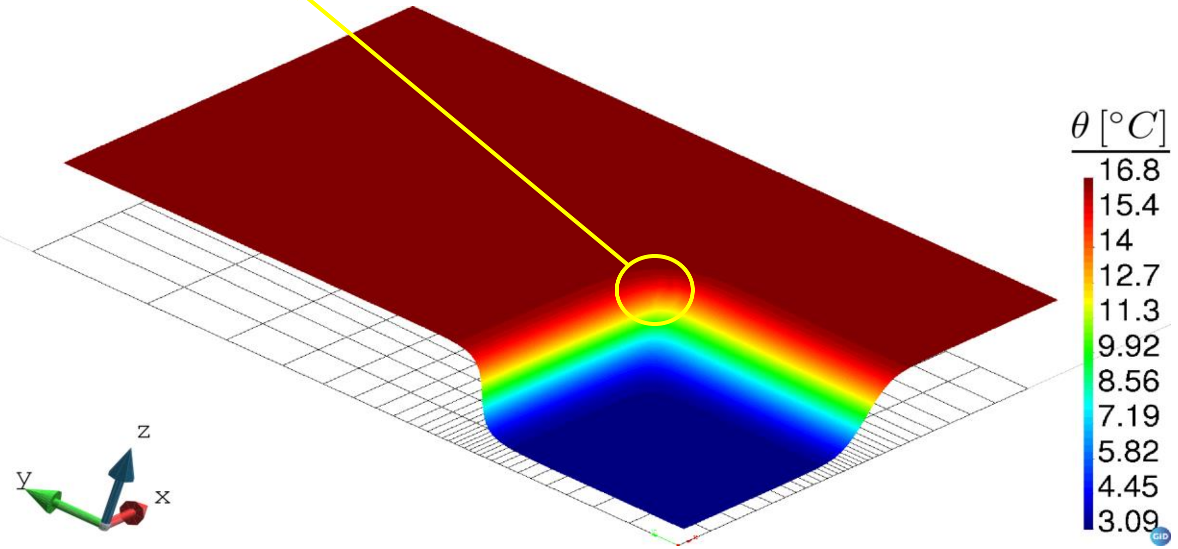
- A demonstrative example of the efficacy of the novel approach in case of non-smooth heating conditions.
- Absence of spurious results at the corner.



Original Biot (mixed) formulation

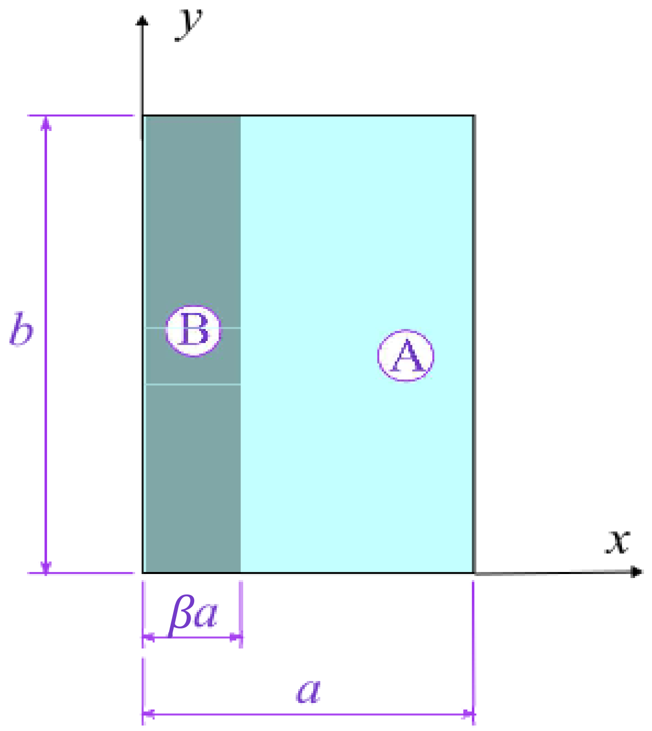


New flux-based formulation

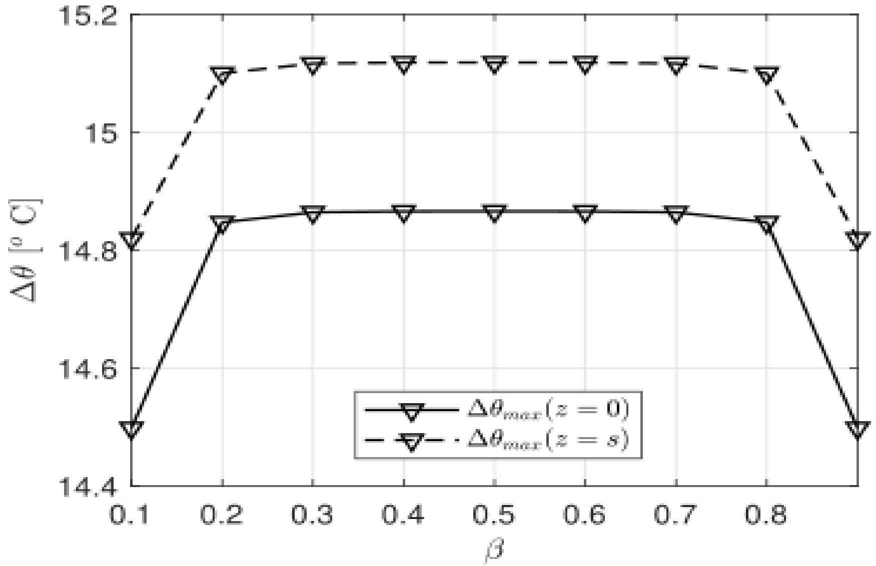


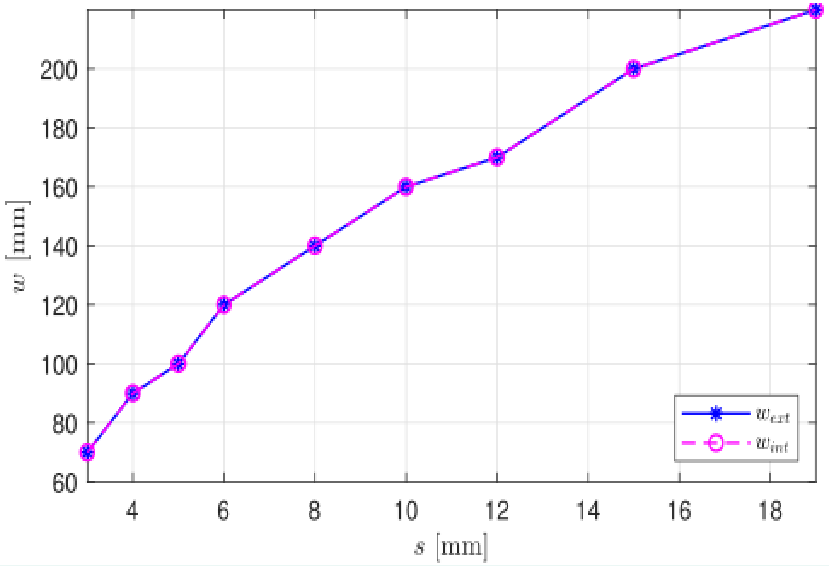
Ex.1: Variable Shadow:

- Considering the effect of changing the shadow area on the thermal response of the pane.

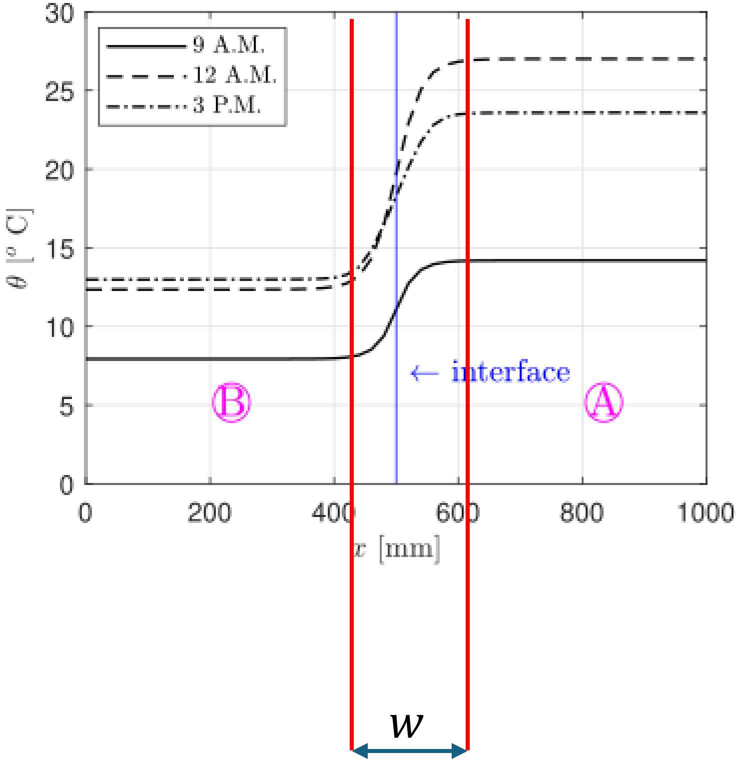


- $\Delta\theta$ as function of shadow width.





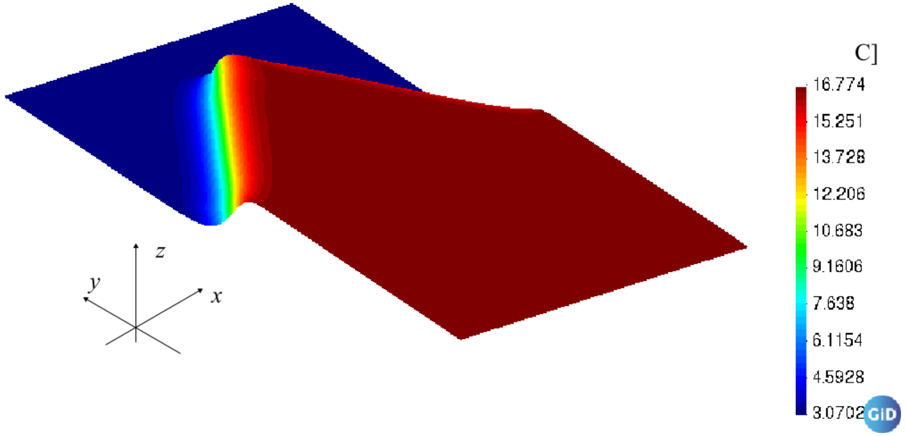
- Sigmoidal transition of temperature profile.

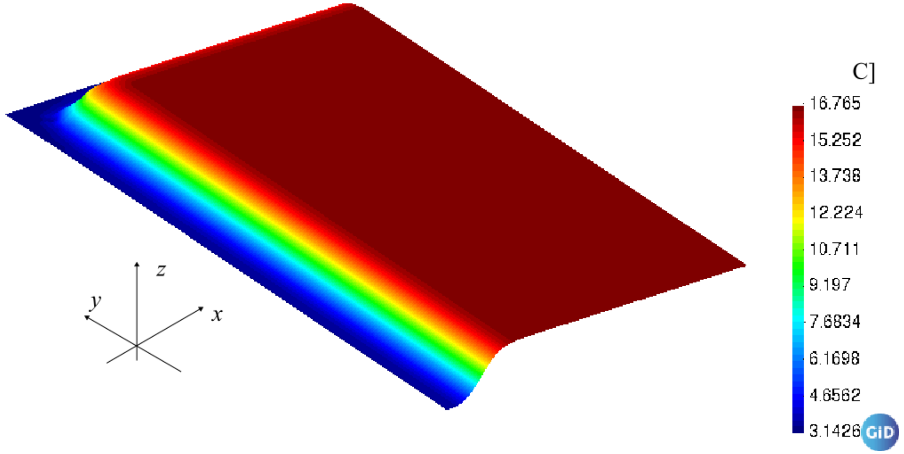


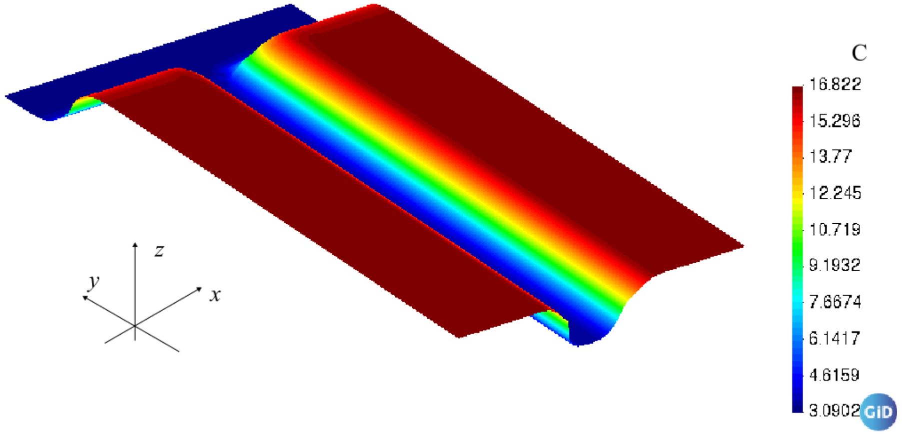
- Width of transition strip depends on plate thickness.

Effect of Shadow Shape

- Considering the effect of different shadow shapes on the thermal response of the pane.
- $\Delta\theta$ between shaded and irradiated area is not affected by shadow shape.

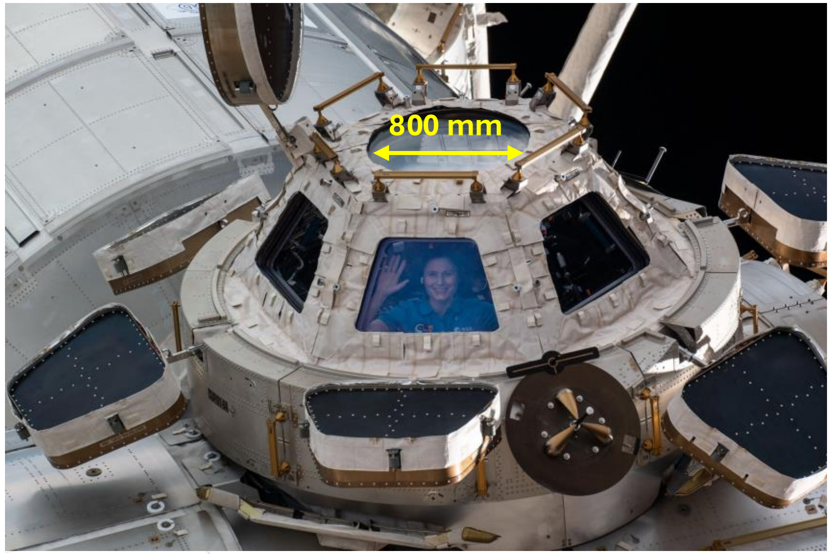


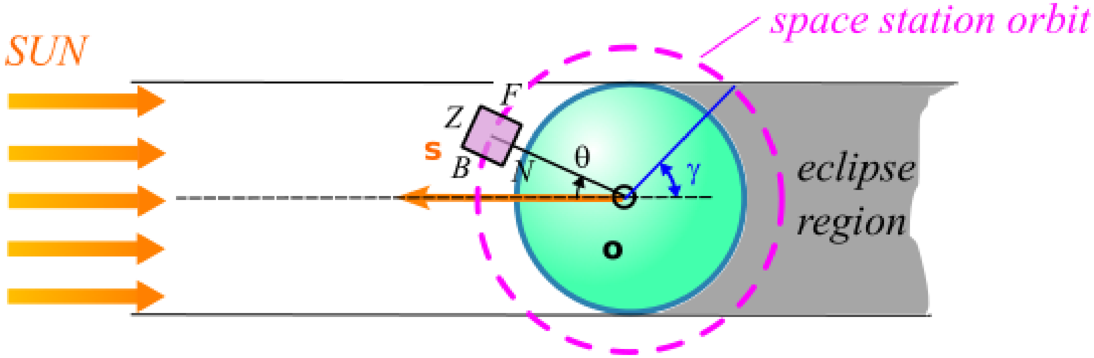


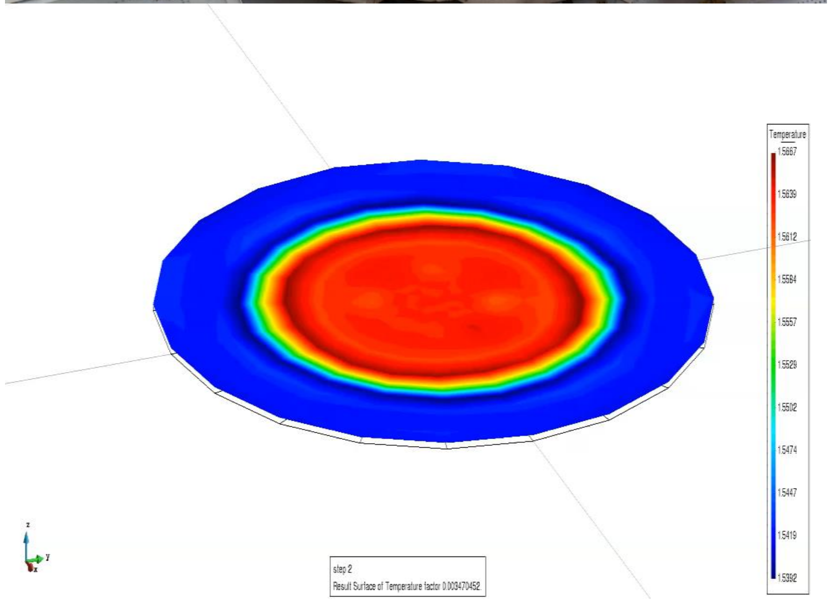


An example in extreme environment. Thermal analysis of ISS cupola window

- Existing layout: Multi layered fused silica panes separated by vacuum.
- Shading due to the supporting frame
- Cyclic exposure to solar radiation



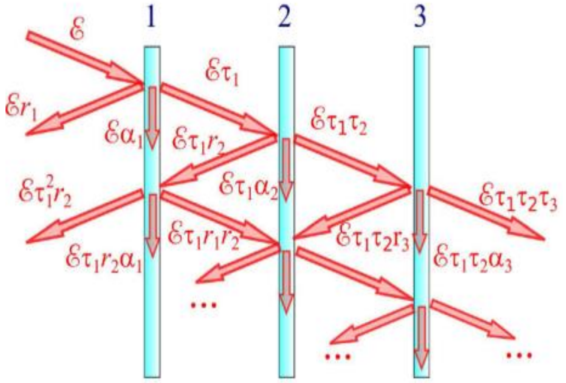


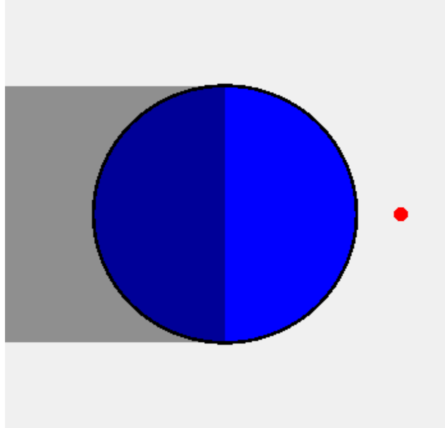


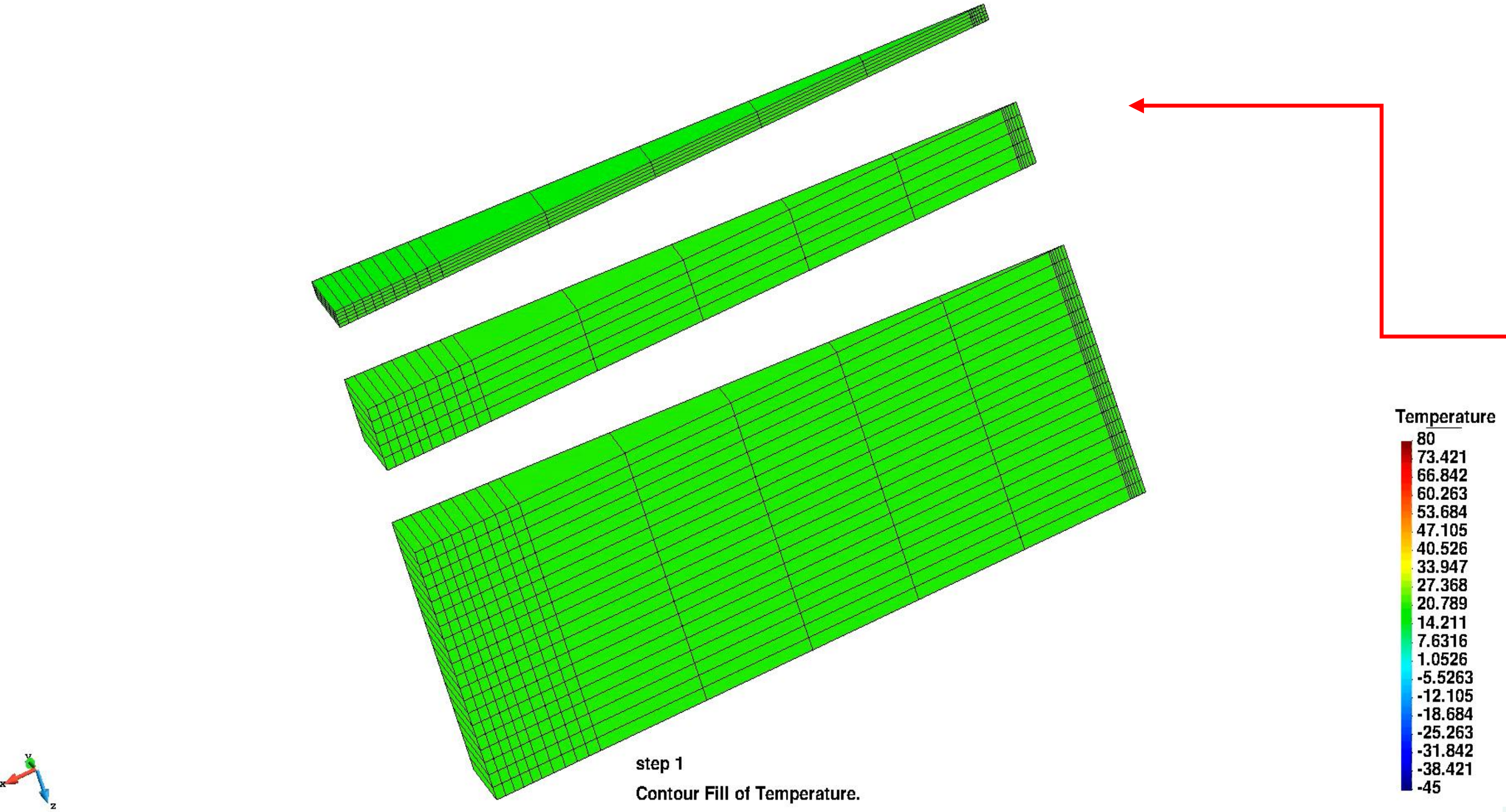
Thermal analysis of ISS copula window

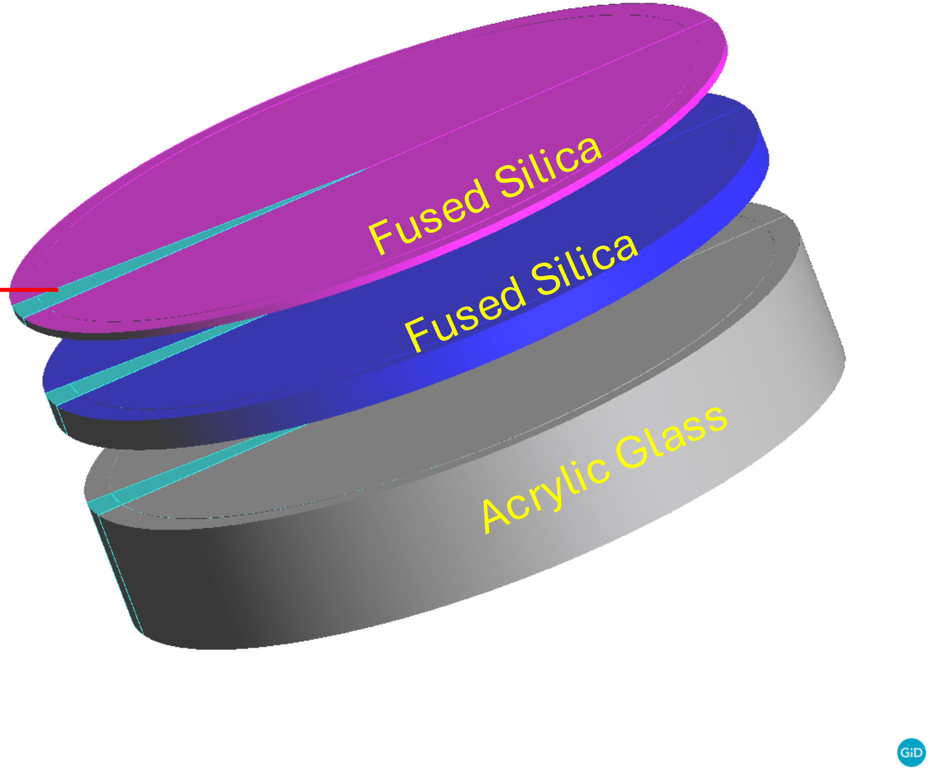
- Considering 3 panes: FS (9.49 mm) + FS (36.83 mm) +AG (150 mm).
- Radial symmetry easily exploited using $\dot{\mathbf{H}}(\mathbf{x}, t) \cdot \mathbf{n} = 0$

Why acrylic glass? Fused silica does not absorb radiation (poor shielding properties). Exploring the possibility of replacing one fused-silica pane.





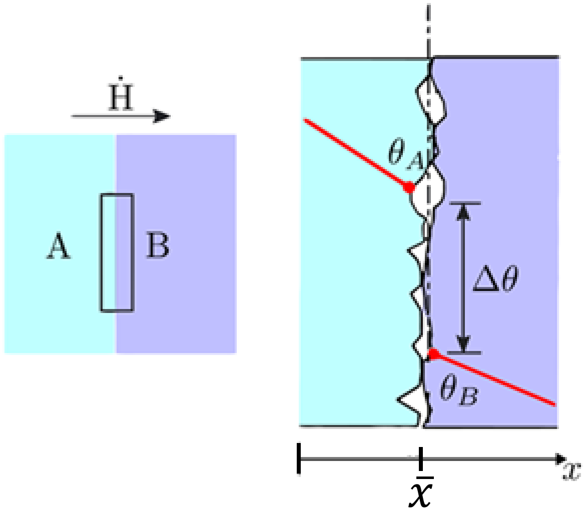




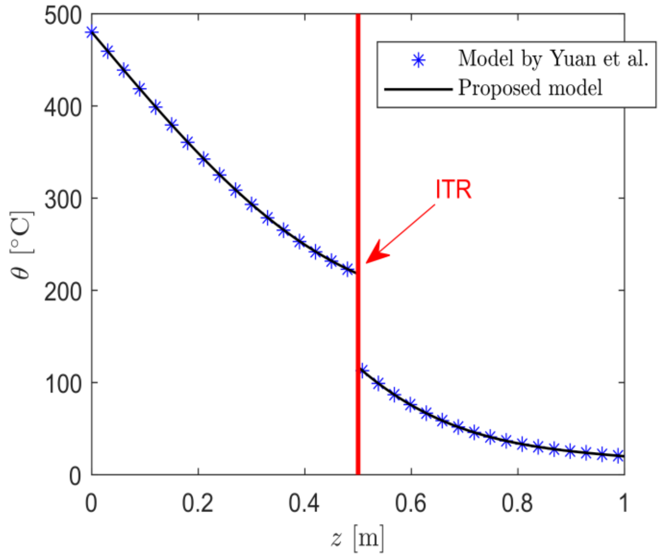
Interfacial Thermal Resistance (Kapitza resistance):

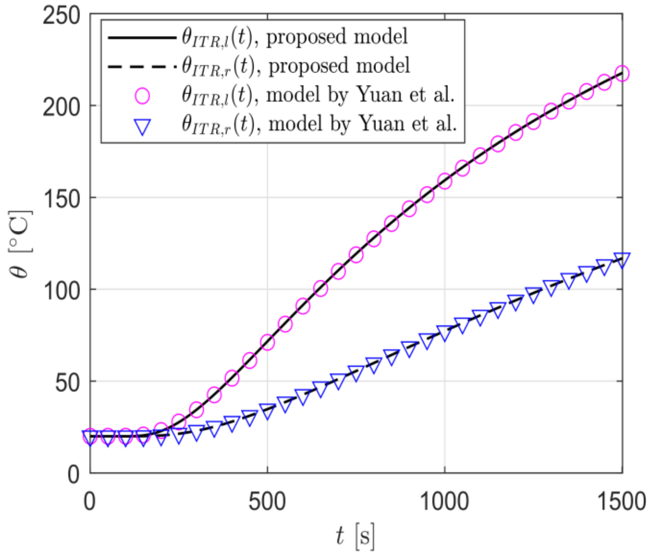
- **Mechanism:** Arises from the scattering of phonons (vibrations in the lattice) and electrons when they attempt to cross the interface between two materials with different properties.
- **Perfect Interface:** Resistance exists even when the interface is atomically perfect and flat.

Solving this problem using the finite difference (FD) method requires dividing the domain into 500 points per layer, whereas the new finite element (FE) approach achieves similar accuracy with only 5 elements per layer.



$$\dot{H}(\bar{x}) = -\frac{1}{R}(\theta_B - \theta_A)$$





The flux-based approach can readily handle this condition

Contributions

- **Novel Neat Variational Principle:** A development of the variational principle originally introduced by Biot in 1955.
- **Simple & Efficient FE Implementation:** Accurate results are obtained with a small number of elements.
- **Advanced Analysis of Laminated Plates and Insulated Glass Units:** transient states, time-varying Shades, multiple reflections, non-smooth temperature field**.**
- **Nonlinear Radiation Absorption Laws:** applicable to material with variable absorption capacity.

If interested in this topic, please send an email to: info@maffeis.it
“I am interested in Thermal analysis of glass facades”