A Flux-based FEM for the Thermal Analysis of Laminated Glass Façades with Cast Shadows

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Abstract

The assessment of the temperature distribution on windows and façade components is crucial for safety and durability of building skins, as thermal stress from solar heating often causes breakage. Cast shadows further enhance temperature irregularities. Approximated methods of analysis, often proposed in standards, are used in the practice to determine the temperature distribution, but these do not always allow for precise calculation of the resulting thermal stress. On the other hand, advanced thermal analysis software is not always accessible to structural engineers. A "flux-based" approach was recently proposed to specifically evaluate the time-dependent 3D temperature distribution in monolithic and layered glazing. This theory expands upon Biot's variational method from the 1950s, which is grounded in the definition of the heat displacement field, whose time derivative represents heat flux. The solar heat absorbed within the material, which can create a complex spatial distribution due to the refractive properties of the materials, is accounted for as an additional component of the heat displacement. The flux-based approach has been implemented in an in-house finite element code, taking advantage of the variational form of the equations. For layered glazing, this framework is in general more efficient than the traditional temperature-based approaches since the variable is the heat displacement, which enjoys a higher regularity than the temperature field. Therefore, it is particularly suitable for problems entailing non-smooth temperatures fields, for example at the interfaces between the different layers of the laminate and between shadowed and exposed regions of the glass surface. As a paradigmatic example, the developed code is used to determine the effect of size and shape of shadows in monolithic and laminated glass panes, obtaining results of general value that can be used to support the use of simplified models for the thermal design of glass façades.

Keywords

Thermal stress, Heat displacement, Glass façades, Finite element analysis, Solar heating

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1. Introduction

The building envelope, which separates the indoor space and contributes to the aesthetic appeal, plays a significant role in energy transfer between the interior and exterior. The glazing system, being the only component capable of capturing direct solar energy due to its transparency, is often regarded as the most critical, especially for the large glass surfaces typically featured in high-end buildings (Galuppi et al, 2023b). A proper thermal design of the glazing elements, accounting for the climatic actions (Wüest & Luible, 2016), namely the time-dependent solar radiation and environmental temperatures, is crucial for safety and durability of building skins. Indeed, uneven temperature distributions may arise, with high gradients aggravated by the shielding from the window frame and projected shadows (Galuppi et al, 2021), possibly leading to glass breakage due to the stress consequent to the differential thermal expansion or, in short, the “thermal stress”. These represent one of the most frequent causes of breakage of building façades (Foraboschi, 2017 and Poláková et al, 2018).

The temperature distribution should be assessed by properly accounting for the time-dependent solar radiation and internal/external temperatures, which are influenced by fluctuating environmental conditions. Additionally, it is important to consider that glazing elements, such as laminated glass or insulating glass units, are composed of different materials, which contribute to irregularities in the temperature field. Approximated methods, commonly found in standards, are usually adopted to estimate temperature distribution, but do not always allow precise thermal stress calculations (Galuppi et al, 2023a). On the other hand, dedicated commercial Software can provide sophisticated analyses but, realistically, can be used only for important projects.

Recently, an innovative approach to evaluate the transient thermal state has been proposed in (Haydar et al, 2024), inspired by the variational formulation for thermal problems originally proposed by M. Biot (1957), and later specialized (Galuppi et al, 2024) to analyze the effects of non-uniform solar radiation on glazing elements caused by cast shadows. The approach is based on the definition of the *heat displacement* field, whose time derivative is the heat flux. Such field enjoys a higher regularity with respect to the temperature field and, therefore, allows to directly consider complex problems.

The flux-based approach has been implemented in an in-house Finite Element (FE) code, leveraging the variational form of the equations (Galuppi et al, 2024). Compared to traditional temperature-based methods, which require high computational effort and fine meshes when dealing with steep gradients, discontinuities, or abrupt temperature jumps, the flux-based approach is generally more efficient, thanks to the higher regularity of the heat displacement field. As a result, it allows for coarser discretization, reducing computational effort and making it particularly suitable for problems with non-smooth temperature distributions, such as in architectural glazing with layered materials and shaded areas. As a case study, the developed code is here used to analyze the effects of shadow size and shape in laminated glass panes, yielding results of general applicability to support simplified thermal design models for glass façades.

2. The thermal problem in architectural glazing

The temperature distribution in a façade panel changes over time, depending on the thermal properties of its various components, such as glass, interlayers, frames, sealants, and coatings. Environmental heating is influenced by factors like the time of day, season, panel inclination, orientation, and the presence of shadows from nearby structures, trees, sunshades, or fins. Consequently, different areas of the panel warm up unevenly, with shaded sections receiving limited solar radiation and the panel edges further insulated by the frame.

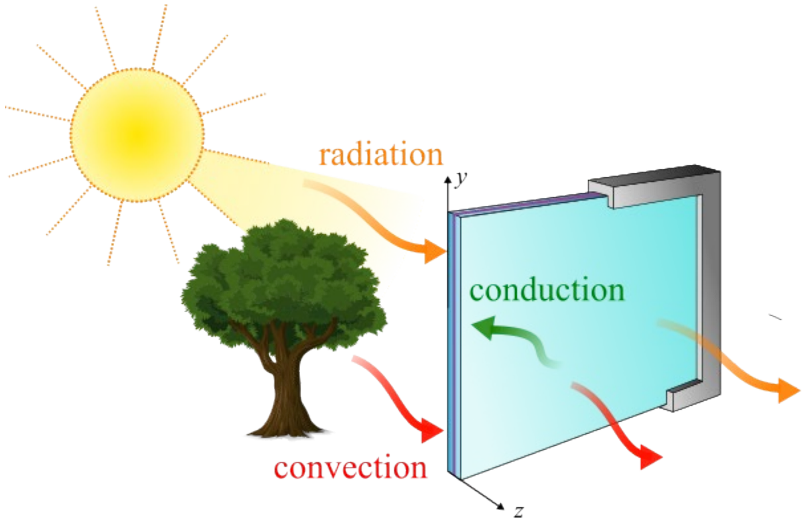


Fig. 1: Schematics of typical thermal scenario and heat exchange phenomena for a façade glazing panel.

The panel's thermal behavior is influenced by various heat exchange mechanisms, as outlined in Figure 1. In particular, in the in-plane directions (x, y) the thermal problem is governed by the heat conduction between different irradiately regions, while in the through-the-thickness direction z, it is dictated by convection and radiation with the surrounding environment, and by conduction between the diverse materials at different temperatures. Furthermore, the glazing retains thermal energy according to its heat capacity.

Determining the temperature field $T(x, y, z, t)$ is crucial to evaluate the thermal stress. Indeed, thermal strains $\varepsilon_T(x, y, z, t)$, which develop when a material is heated or cooled with respect to a reference temperature T_0, are dictated by the temperature according to

$$\boldsymbol{\varepsilon}_T(x, y, z, t) = \alpha_T[T(x, y, z, t) - T_0]\,\boldsymbol{I}, \tag{1}$$

where α_T is the coefficient of thermal expansion and $\boldsymbol{I}$ is the identity tensor. The thermal stress tensor reads

$$\boldsymbol{\sigma}(x, y, z, t) = \boldsymbol{C}[\boldsymbol{\varepsilon}(x, y, z, t) - \boldsymbol{\varepsilon}_T(x, y, z, t)], \tag{2}$$

where $\boldsymbol{C}$ is the constitutive elastic fourth-order tensor and $\boldsymbol{\varepsilon}(x, y, z, t)$ is the total strain tensor, accounting for both the thermal and the mechanical parts of the deformation, a priori unknown.

2.1. The climatic actions

The total density of heat flow rate of incident solar radiation, usually denoted as *G(t)*, depends on several factors, such as the season and time of day, façade orientation and geographic location, and panel inclination (Zhang et al, 2017). The standards (AFNOR, 2006) usually record values for $G(t)$ in tables and graphs as a function of the season and the orientation and inclination of the façade. For a winter scenario, the time dependence of $G(t)$ can be approximated (Galuppi et al, 2021) with the parabolic law shown in Fig. 2. The amount of heat absorbed by each layer composing the glazing is dictated by reflectivity, transmissivity and absorptivity of the different plies composing the package (Galuppi & Royer-Carfagni, 2022).

At the external and internal surfaces, the element exchanges heat by convection with the surrounding air, at temperatures $T_{int}(t)$ and $T_{ext}(t)$ respectively. While the former is usually assumed to be constant in time, the time dependence of the latter can be approximated with a sinusoidal law, of the type shown in Fig. 2 for a standard winter scenario (AFNOR, 2006). Furthermore, the external surface of a glazing is irradiated by the sky vault, as well as by other exterior surfaces, which can be treated as large enclosure surfaces at a temperature $T_{sky}(t)$, possibly time-dependent, as well as with the internal room surfaces, assumed at the same temperature of the internal air (ISO TC163 /SC2, 2003).

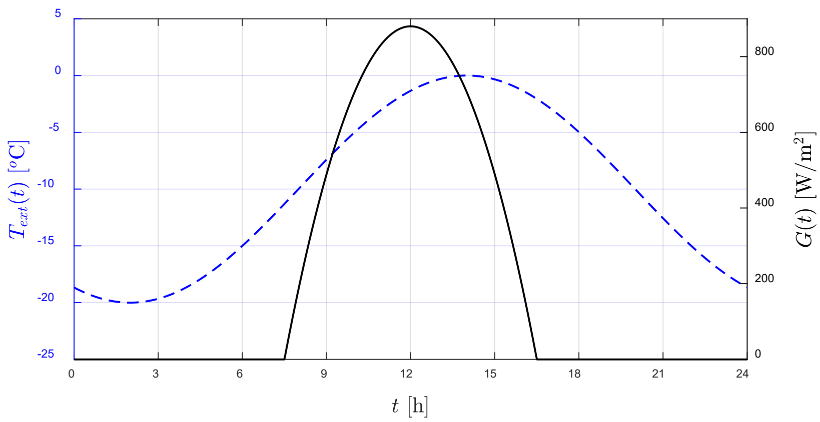


Fig. 2: Daily variation of solar radiation $G(t)$ and external temperature $T_{ext}(t)$, for a winter scenario.

2.2. The governing equations

The thermal problem, expressed in terms of the time-dependent temperature field $T(x, y, z, t)$, is governed by the classical heat conduction equation

$$\rho c_p \frac{\partial T(x,y,z,t)}{\partial t} = \lambda \left[\frac{\partial^2 T(x,y,z,t)}{\partial x^2} + \frac{\partial^2 T(x,y,z,t)}{\partial y^2} + \frac{\partial^2 T(x,y,z,t)}{\partial z^2}\right] + q^{\#}(x, y, z, t) \quad (3)$$

Here, ρ represents the material density, c_p denotes the specific heat capacity, λ corresponds to the thermal conductivity, and $q^{\#}(t)$ is the absorbed heat per unit volume, due to the solar heating $G(t)$, which can create a complex spatial distribution in the glass. In simple terms, Eq. (3) states that the thermal energy stored in the glass per unit time and unit volume (expressed by the left-hand side) must equal the combined effect of heat transfer through conduction and the contribution from the absorbed solar radiation. The heat flux vector field is defined as

$$\dot{\boldsymbol{H}}(x, y, z, t) = \begin{bmatrix} \dot{H}_x(x, y, z, t) \\ \dot{H}_y(x, y, z, t) \\ \dot{H}_z(x, y, z, t) \end{bmatrix} = -\lambda \begin{bmatrix} \partial T(x, y, z, t)/\partial x \\ \partial T(x, y, z, t)/\partial y \\ \partial T(x, y, z, t)/\partial z \end{bmatrix} \quad (4)$$

The boundary conditions for the panel surface, in contact with the external ($z = 0$) and internal ($z = s$) environment, account for both radiant and convective heat exchange, and can be expressed as

$$\dot{H}_z|_{z=0} = -\lambda \frac{\partial T}{\partial z}|_{z=0} = \varepsilon\sigma\left[T_{sky}^4 - T^4|_{z=0}\right] + h_{e;c}\left[T_{ext} - T|_{z=0}\right] \tag{5a}$$

$$\dot{H}_z|_{z=s} = -\lambda \frac{\partial T}{\partial z}|_{z=s} = \varepsilon\sigma\left[T^4|_{z=s} - T_{int}^4\right] + h_{i;c}\left[T|_{z=s} - T_{int}\right] \tag{5b}$$

where the dependence of temperature and heat fluxes on (x, y, z, t) is omitted for the sake of brevity; σ is the Stefan-Boltzmann constant, ε is the material emissivity, while $h_{e;c}$ and $h_{i;c}$ are the convective heat transfer coefficient with the external and internal environment, respectively.

Notice that, for standard architectural glazing, the panel temperature is usually quite close to both the internal and the external temperatures (more specifically, the temperature difference is small with respect to the mean absolute temperature). Therefore, the boundary conditions (5) can be linearized in the form

$$\dot{H}_z|_{z=0} = -\lambda \frac{\partial T}{\partial z}|_{z=0} \cong h_e[T_{ext}^* - T|_{z=0}] \tag{6a}$$

$$\dot{H}_z|_{z=s} = -\lambda \frac{\partial T}{\partial z}|_{z=s} \cong h_i\left[T|_{z=s} - T_{int}\right] \tag{6b}$$

where h_e and h_i are the total heat transfer coefficient with the external and internal environment, respectively, while T_{ext}^* is a fictitious temperature, defined according to (Galuppi & Royer-Carfagni, 2022). For more complicated problems, as for example for glazing used in space applications, the full fourth-order boundary conditions (5) must be retained.

For laminated glass elements, the thermal problem is complicated by the presence of various layers having different thermal properties (ρ, c_p and λ). In this case, the heat-conduction Eq. (3) holds for each of the plies composing the glazing, while proper interface conditions (i.e., the continuity of the temperature and of the heat flux) should be required at the interfaces between different materials.

2.3. The flux-based approach

The ''neat'' flux-based variational approach for thermal problems (Haydar et al., 2024) is based on Biot's definition (Biot, 1957) of the *heat displacement* vector field $\mathbf{H}(x, y, z, t)$, whose time derivative is the heat flux of Eq. (4). It was shown in (Haydar et al., 2024) that a variational principle can be formulated, correspondent to the governing equation of the thermal problem (3), in terms of $\mathbf{H}(x, y, z, t)$ only. This approach can be implemented in a FE code, by dividing the domain in finite elements, in each one of which the material is homogeneous. The (time-dependent) degrees of freedom of the problem are the nodal heat displacements; tri-linear shape functions provide a reasonable interpolation between the nodal points. Denoting with $\tilde{\mathbf{H}}\,(t)$ the vector collecting the heat displacement nodal values, the discretized form of the equation governing the problem in terms of the heat displacement field $\mathbf{H}(x, y, z, t)$, can be written in the matrix form

$$\widetilde{\boldsymbol{H}}(t) + \boldsymbol{C}\dot{\widetilde{\boldsymbol{H}}}(t) = \boldsymbol{q}(t) + \boldsymbol{q}_0 - \boldsymbol{K}\,\widetilde{\boldsymbol{H}}^{\#}(t) \tag{7}$$

where $\mathbf{K}$ and $\mathbf{C}$ are global "stiffness" and "damping" matrices; $\mathbf{q}(t)$ and $\mathbf{q}_0$ indicate the contributions respectively due to the (possibly time-dependent) temperature distribution at the boundaries, and the initial temperature distribution; the last term accounts for the solar heat absorbed within the material. Once the heat flux is evaluated by solving Eq. (7), the Eq. (4) is used *a posteriori*, to recover the temperature field.

Compared to standard temperature-based numerical formulation, the flux-based approach offers greater efficiency. Indeed, the temperature field is not explicitly present in Eq. (7), so that the continuity of the temperature field is not enforced; hence, the model can be easily used to handle problems with temperature discontinuities, or with not-smooth temperature profiles. On the other hand, in the classical approaches entailing the discretization of Eq. (3), it is necessary to know in advance the potential surface of discontinuity for the temperature. Moreover, since boundary conditions (5-6) entail both the value of the boundary temperature and its normal derivative, standard approaches require the use of high order shape functions, and/or of a very fine mesh. Such approaches can provide numerical errors in the presence of very steep temperature gradients, which often occur façade glazing elements.

3. Examples

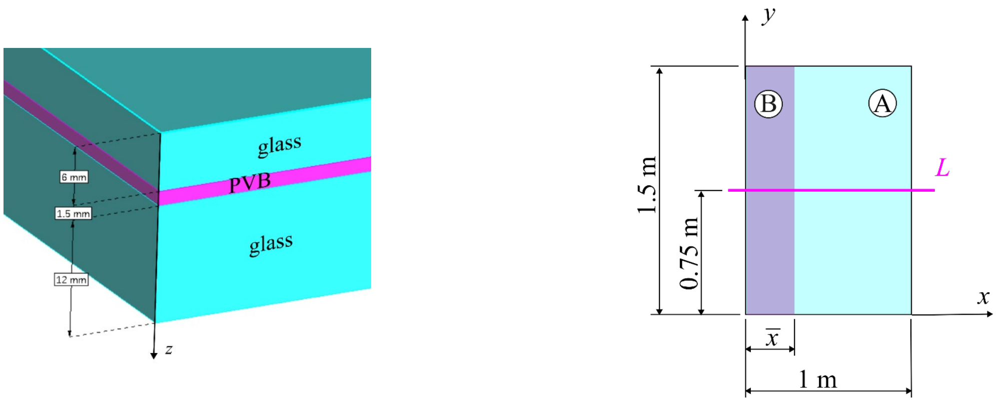
The considered model problem is that of a rectangular laminated glass element, of size 1 m × 1.5 m, composed of two glass plies of thickness 6 mm (on the external side) and 12 mm, bonded by a PVB interlayer of thickness 1.5 mm, as shown in Fig. 3(a). In the sequel, uneven radiation conditions on the glass surface, produced by external shadowing elements with different sizes and shapes are considered. Values for the thermal properties for glass and PVB used in the analyses are collected in Table 1. The internal and external heat transfer coefficients used in boundary conditions (6) are h_i = 8.375 W/(m2K) and h_e = 11.926 W/(m2K), respectively.

Table 1: Values of thermal properties of materials considered in the analyses.

Material	ρ [kg/m³]	c_p [J/(kg K)]	λ [W/(m K)]
Glass	2500	720	1
PVB	1087	1370	0.236

3.1. Partially shaded panel

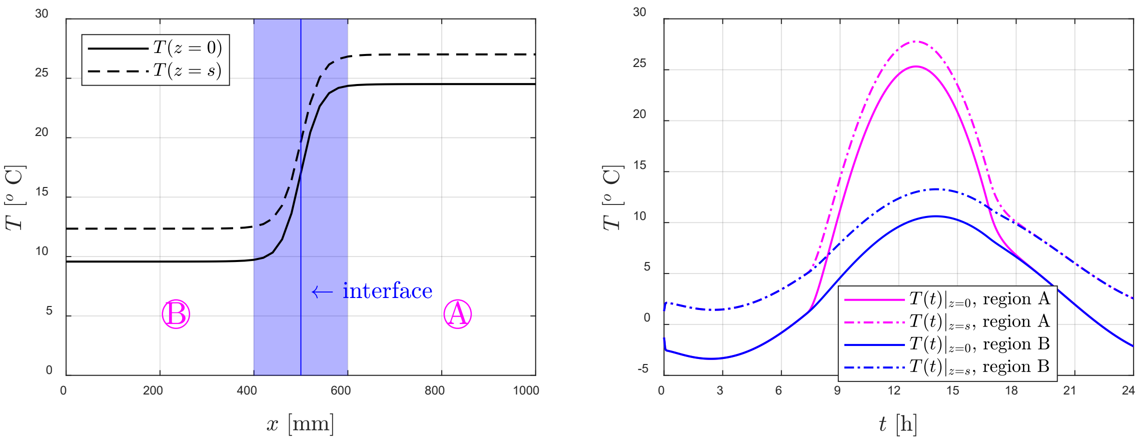
To investigate the influence of the size of the shaded area, consider the rectangular laminated glass pane of Fig. 3(b), with a projected shadow in region B, of width $\bar{x} \in [0, a]$, and fully irradiated in region A. For a winter scenario, the solar radiation $G(t)$ and the external temperature $T_{ext}(t)$ are made to vary according to daily variations shown in Fig. 2.



a) b)

Fig. 3: (a) Considered laminated glass package and (b) model problem of the shaded panel.

For $\bar{x}$ = 0.5 m, Fig. 4(a) show the spatial temperature distribution along the line *L* of Fig. 3(b), at 12 A.M., at the external (z = 0) and internal (z = s) surfaces. Observe that this is uniform in the two regions, except in a strip across the interface between them, whose width is of the order of 0.2 m, where the temperature profile has a sigmoidal trend. Fig. 4(b) shows the time evolution along a day of the temperature at the panel surfaces, evaluated at points of regions A and B sufficiently far from the interface, where the temperature is substantially uniform. The time-evolution of the temperature field is influenced by the parabolic variation of $G(t)$ and the sinusoidal trend of $T_{ext}(t)$.



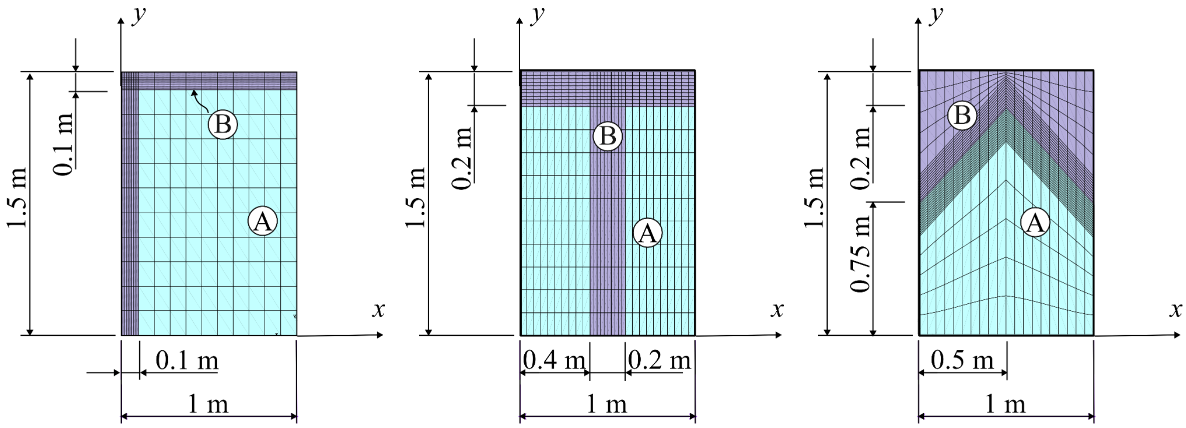
a) b)

Fig. 4: (a) Spatial temperature distribution along the line L of Fig. 3(b), at 12 A.M., and (b) time evolution of the temperature on the panel surfaces.

It was observed in (Galuppi et al, 2024) that the temperatures in regions A and B, as well as their difference, are not significantly influenced by the distance $\overline{x}$. Additionally, the transition region was found to have a width approximately 15 times the plate thickness. These findings suggest that assessing the maximum temperature difference does not require to analyze every possible configuration, as the temperature variation is only minimally impacted by the shadow's size.

3.2. Influence of the shadow shape

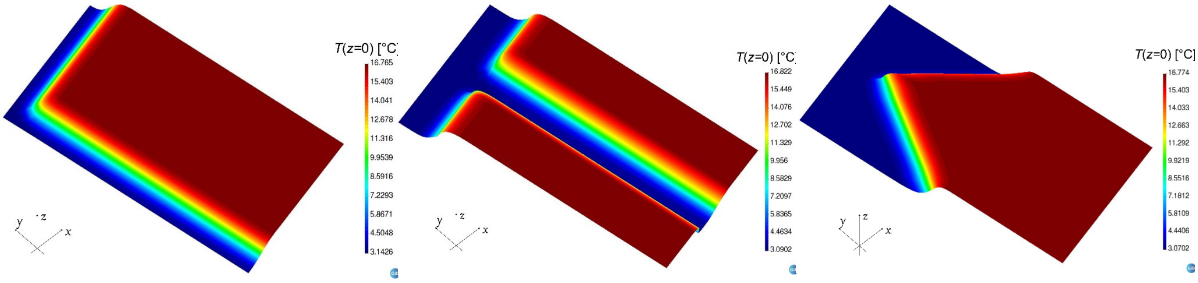
To investigate the influence of the shadow shape, consider the various shadow geometries illustrated in Fig. 5, together with the mesh used in the FE simulations, selected based on the qualitative risk classification mentioned in (CNR, 2013). To investigate the temperature distribution, now fixed environmental conditions, with $G(t)$= 800 W/m^2 and $T_{ext}(t)$= $T_{sky}(t)$ = -12°C, are considered.



a) b) c)

Fig. 5: Different shadow shapes considered in the study: (a) L-shaped, (b) T-shaped and (c) V-shaped.

Under constant environmental conditions, the glazing panel attains a steady-state condition within approximately two hours. The resulting temperature distribution on the external surface of the panel, corresponding to different shadow shapes, is depicted in Fig. 6. A qualitatively similar distribution is observed on the internal surface.



a) b) c)

Fig. 6: Steady-state temperature distribution in correspondence of the external surfaces for (a) L-shaped, (b) T-shaped and (c) V-shaped shadows.

Notably, the temperatures in the uniform sections of regions A and B are identical in the three considered cases. Additionally, the transition between these regions follows the same sigmoidal pattern shown in Fig. 4, i.e., it does not depend on the shape of the shadow. However, this does not imply at all that the thermal stress is independent of the shape and size of the shadows, since this is certainly influenced by possible sources of concentrations, as re-entrant corners, in the shadow shape.

4. Conclusions

A in-house Finite Element code, based on an innovative flux-based variational formulation, has been proposed for the thermal analysis of laminated glass façades with cast shadows. The approach formulates the thermal problem in terms of the *heat displacement field*, enjoying a much high regularity with respect to the temperature field. Environmental temperatures only appear in boundary and initial conditions, while glass temperature can be computed a posteriori. Compared to traditional formulations based on the discretization of the temperature field, this flux-based method offers greater numerical efficiency and simplifies the definition of boundary and interface conditions compared to traditional heat conduction formulations. Therefore, it seems to represent the ideal tool to model laminated glass elements with projected shadows, for which the irregularity of the temperature distribution can cause high thermal stresses in glass, representing one of the most frequent causes of breakage of building façades.

A specific application of the code has regarded the effect of size and shape of cast shadows on the temperature distribution. We demonstrated that, in laminated glass for architecture, the temperatures in the shaded and irradiated regions are almost independent on their size and shape, and that the width of the transition region, where the temperatures connect with a sigmoidal profile, is of about 15 times the panel thickness. This finding is of paramount importance, as simplified models like those proposed by Galuppi et al. (2021, 2023b), enable reasonably accurate estimation of temperatures at a distance from the interface. By linking these temperatures through a sigmoidal profile in the transition zone, the entire temperature field can be defined. Once the temperature field, and consequently the thermal strain, are known, a commercial FEM code can be used to compute the thermal stress. However, for more complex scenarios, in which the temperature gradients need to be precisely calculated, e.g. for the glazing used in space applications, the proposed flux-based FE approach is a valuable tool to precisely assess the temperature profile within the transition zone.

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